Side plates in dowel shear connections: tests and analysis

Background

This paper complements "the mathematics and computer modeling of nailed shear connections", and "timber connections with mixed properties in single and double shear". The former gives shows how the basic equations were derived from theory and tests, and the latter gives a series of worked examples.

Although the derivation of the ***EDM*** is complex, "suggested code wording" shows the extreme simplicity of its application.

Note that the "crushing stress" is constant regardless of dowel diameter.

Equations

The test results derived by Andre Jorissen in 1998 for a series of dowelled shear connections in double shear are shown in Figure 1 and compared with predictions by two very different analytic models.

The first is that made by the European Yield Model (***EYM***) which represents the pressure exerted by the wooden matrix upon the steel dowel as a series of uniform rectangular distributions in accordance with the Johansen equations.

The second is given by the Exponential Distribution Model (***EDM***) which uses triangular or trapezoidal distributions of pressure and, besides providing a much simpler means of prediction than the ***EYM,*** permits accurate analysis which is not possible with the ***EYM.***

The ***EYM*** results are based upon use of the ***bedding*** stress (F_b) and the ***EDM*** uses the ***crushing*** stress (F_c). Both stresses are based upon tests ; extensive in the case of the ***EYM*** and limited in the case of the ***EDM.*** The crushing stress (F_c) is about one and a third times the magnitude of the ultimate bedding stress (F_b)

The ***EYM*** is based upon consideration of the shear interface(s), whereas the ***EDM*** equations consider the resistance of the individual elements comprising the connection. In Figure 1 L_s is the thickness of the side plate and L_m the thickness of the centre plate. l is the critical embedment calculated from

$2.12\sqrt{\frac{F_y}{F_c}} \times d$, (***Equation 1***) and r is the embedment (reduction) factor applied to the ultimate shear for each element. The embedment factor r for side plates L_s is $\sqrt{\frac{L_s}{l}}$, and for main members L_m is $\frac{L_m}{l}$.

In the ***EDM*** the ultimate shear which an element (with embedment equal to or greater than l), will withstand in a test is given by $V_{ult} = .53\sqrt{F_y F_c}.d^2$ (***Equation 2***).

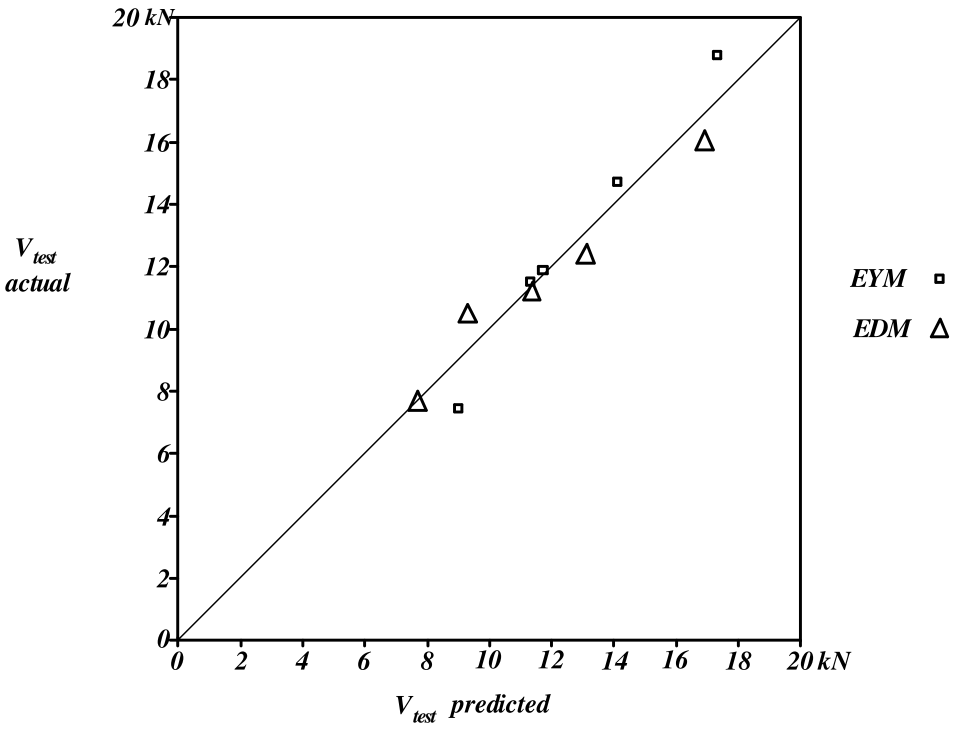
Both l and V_{ult} are constant in both equations for values of F_y , F_c and d the dowel diameter. In Figure 1 F_y is taken as 400 MPa (680/1.7), and F_c is 43 MPa for spruce. The predicted values in Figure 1 for the ***EYM*** are taken from Andre Jorissen's paper except in Series 1where he used a fracture mechanics prediction .

None of the connections tested by Andre Jorissen in 1998 provided adequate embedment. The intent of the testing was to examine the behaviour of such connections in double shear

The ***EDM*** results show side plate values in the tabulation compared with a test load which causes a hinge to develop at the centre line (shown as "***EDM Analysis"*** in the tabulation).

Analysis shows that side plates govern in double shear connections.

Figure 1 : Test Results



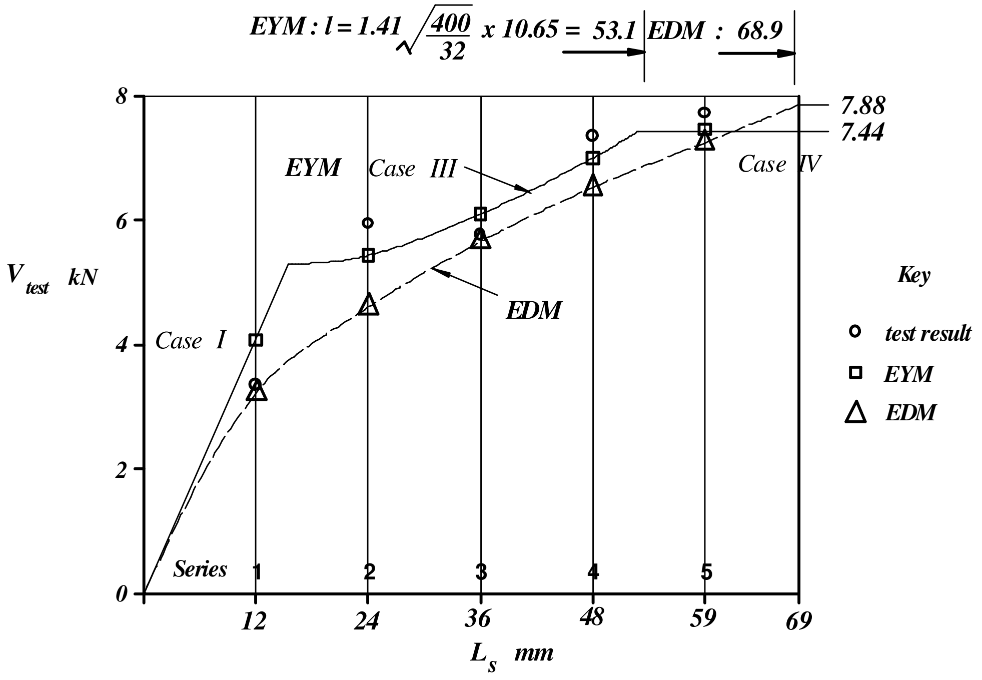
Series	1		2		3		4		5	
d mm	11.75		10.65		10.65		10.65		11.75	
L mm	L_s	L_m	L_s	L_m	L_s	L_m	L_s	L_m	L_s	L_m
	12	24	24	48	36	48	48	48	59	72
l mm	75.9		68.9		68.9		68.9		75.9	
r	$\sqrt{\frac{12}{75.9}}$	$\frac{24}{75.9}$	$\sqrt{\frac{24}{68.9}}$	$\frac{48}{68.9}$	$\sqrt{\frac{36}{68.9}}$	$\frac{48}{68.9}$	$\sqrt{\frac{48}{68.9}}$	$\frac{48}{68.9}$	$\sqrt{\frac{59}{75.9}}$	$\frac{72}{75.9}$
	.40	.32	.59	.70	.72	.70	.83	.70	.88	.95
V_{ult} kN	9.60		7.88		7.88		7.88		9.60	
$V_{ult} \times r$	3.84	3.07	4.65	5.52	5.67	5.52	6.54	5.52	8.45	9.12
EDM $V_{ult} \times r \times 2$	7.68		9.30		11.34		13.08		16.9	
EDM Analysis	7.68		10.5		11.2		12.4		16.0	
EYM Equations	9.02		11.3		11.7		14.1		17.3	
EYM Test	7.46		11.9		11.5		14.7		18.8	

Figure 2 : Test Results for Side Plates (normalized)

Figure 2 serves several purposes. First, it was used to derive the expression ***r*** for side plates**.** For this purpose the results are normalized to a dowel diameter of 10.65 mm.

Note the derivation of critical embedment and ultimate shear load shown on the graph, and how the complexity of the ***EYM*** method for dealing with the various cases *I, III,* and *IV* is simplified in the ***EDM*** where cases *I* and *III* are replaced by the square root reduction of the case *IV* load.

Second, it clearly illustrates the differences between the ***EYM*** and the ***EDM*** **.**The ***EYM*** values tabulated in Figure 2 are not taken from Jorissen's paper but calculated as shown on page 6.



Series	1	2	3	4	5
d mm	11.75	10.65	10.65	10.65	11.75
d normalised	10.65	10.65	10.65	10.65	10.65
norm. factor	.906	1.0	1.0	1.0	.82
L_s	12	24	36	48	59
EDM :	$l = 2.12\sqrt{\frac{F_y}{F_c}} \times d = 2.12\sqrt{\frac{400}{43}} \times 10.65 = 68.9$ $r = \sqrt{\frac{L_s}{l}} = \sqrt{\frac{L_s}{68.9}}$				
r	.417	.590	.723	.835	.925
EDM :	$V_{ult} = .53\sqrt{Fy.F_c} \cdot d^2 = .53\sqrt{400 \times 43} \times 10.65^2 = 7.88$ kN				
$V_{ult} \times r$	3.28	4.64	5.70	6.58	7.29
Test	3.38	5.95	5.75	7.35	7.71
EYM	4.09	5.42	6.09	7.0	7.44

Figure 2

EYM per Shear Plane : Side member : governing case underlined.

In Figure 2 the plot of ***EYM*** values was derived as follows :

Case III

$$R_K = \frac{f_b.t_2.d}{2+\beta} \times \left[\sqrt{2\beta(1+\beta) + \frac{4\beta(2+\beta)M_y}{f_b.d.t_2^2}} - \beta \right]$$

$$\beta = 1 \ , \ f_b = F_b \ , \ M_y = F_y \times \frac{d^3}{6}, \quad R_K = V_{ult} \ , \ t_2 = L_s$$

$$V_{ult} = \frac{F_b.d.L_s}{3\times 10^3} \times \left[\sqrt{4 + \frac{2.F_y}{F_b} \times \left(\frac{d}{L_s} \right)^2} - 1 \right] \text{ kN}$$

$$F_b = 32 \text{ MPa} \ , \ d = 10.65 \text{ mm} \ , F_y = 400 \text{ MPa}$$

Series	1	2	3	4	5
L_s	12	24	36	48	59
V_{ult}	5.27	$\underline{5.42}$	$\underline{6.09}$	$\underline{7.00}$	8.00

Case I

$$V_{ult} = F_b \times d \times L_s$$

$$= 32 \times 10.65 \times 12 \Big/ 10^3$$

$$= \underline{4.09}$$

Case IV

$$V_{ult} = .58\sqrt{F_y F_b} \times d^2$$

$$= .58\sqrt{400 \times 32} \times 10.65^2 \Big/ 10^3$$

$$= \underline{7.44}$$

Analysis

Whereas ***equations 1*** and ***2*** of the ***EDM*** give good approximations of test results, a more accurate evaluation can be made by using a beam analysis program with triangular or trapezoidal loading diagrams on the various elements.

Figure 3 : Analysis of Series 1

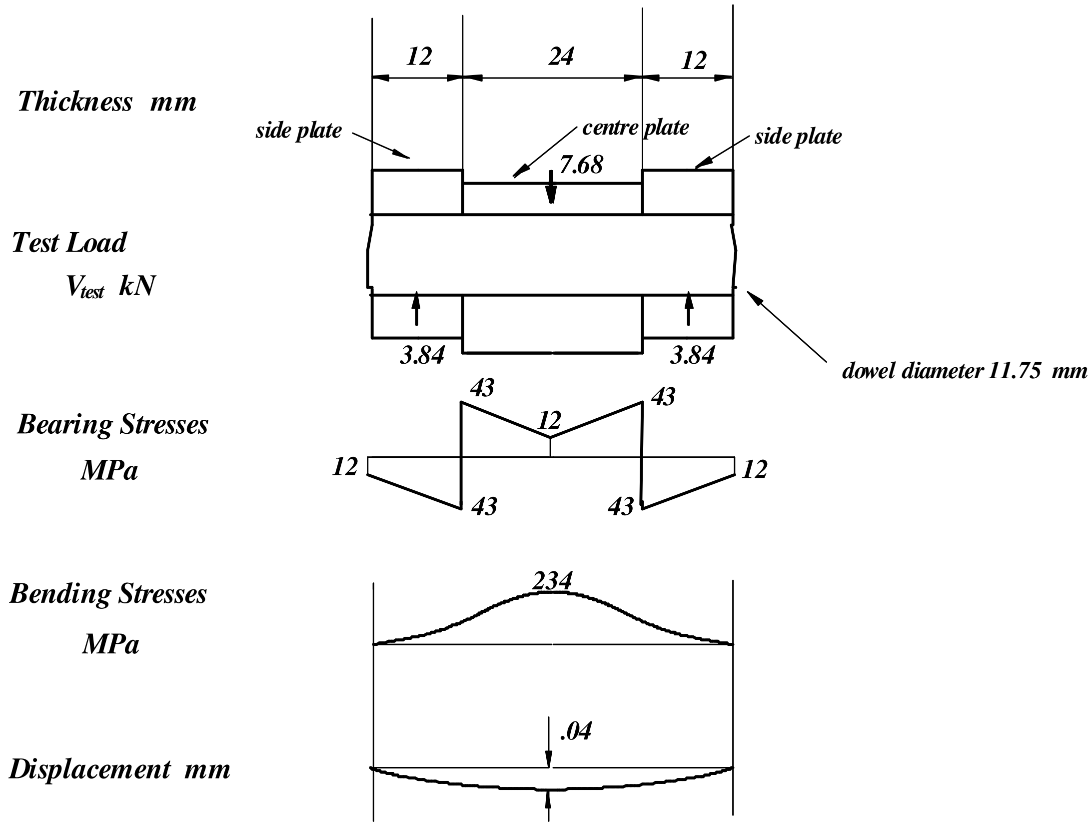


Figure 4 : Analysis of Series 2

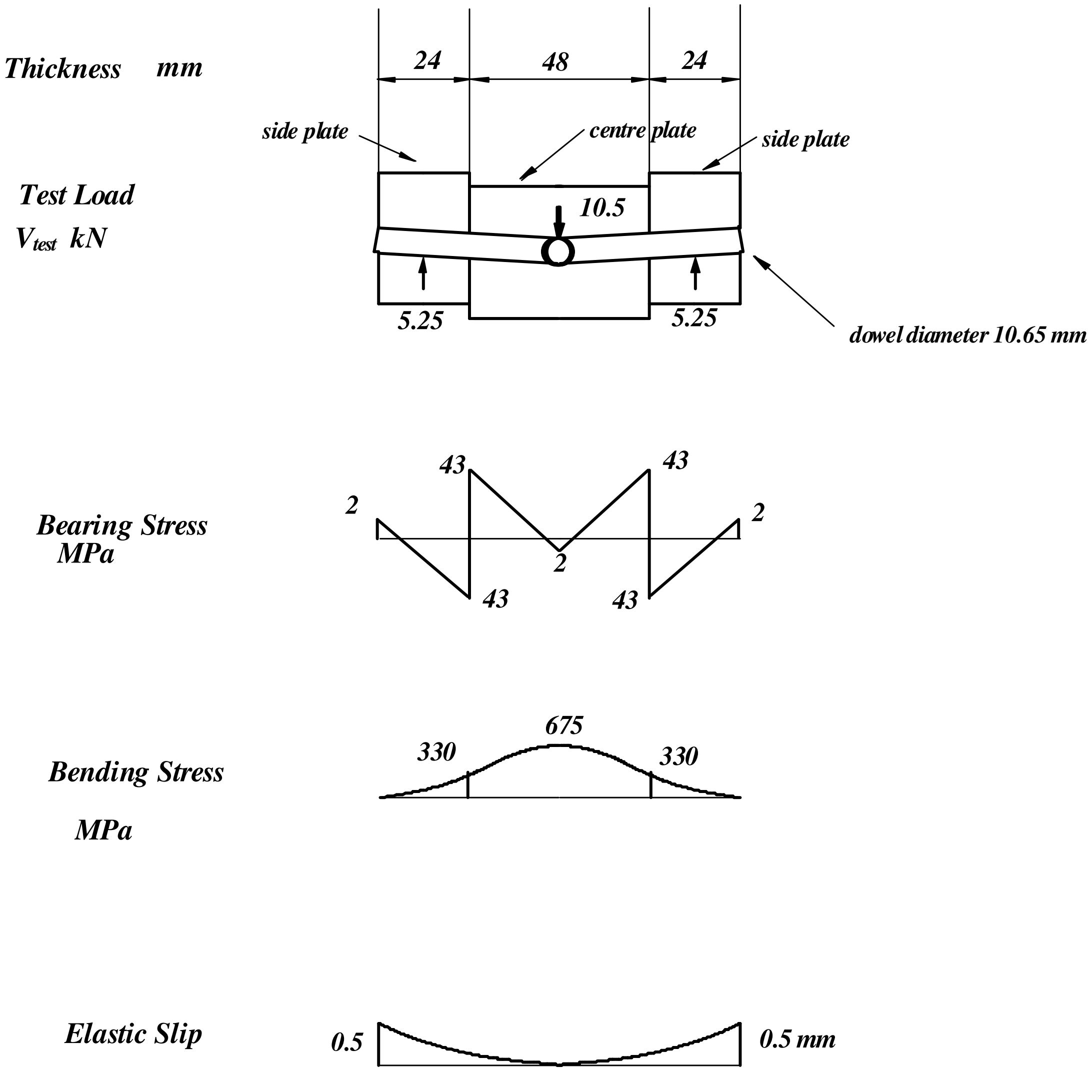


Figure 5 : Analysis of Series 3

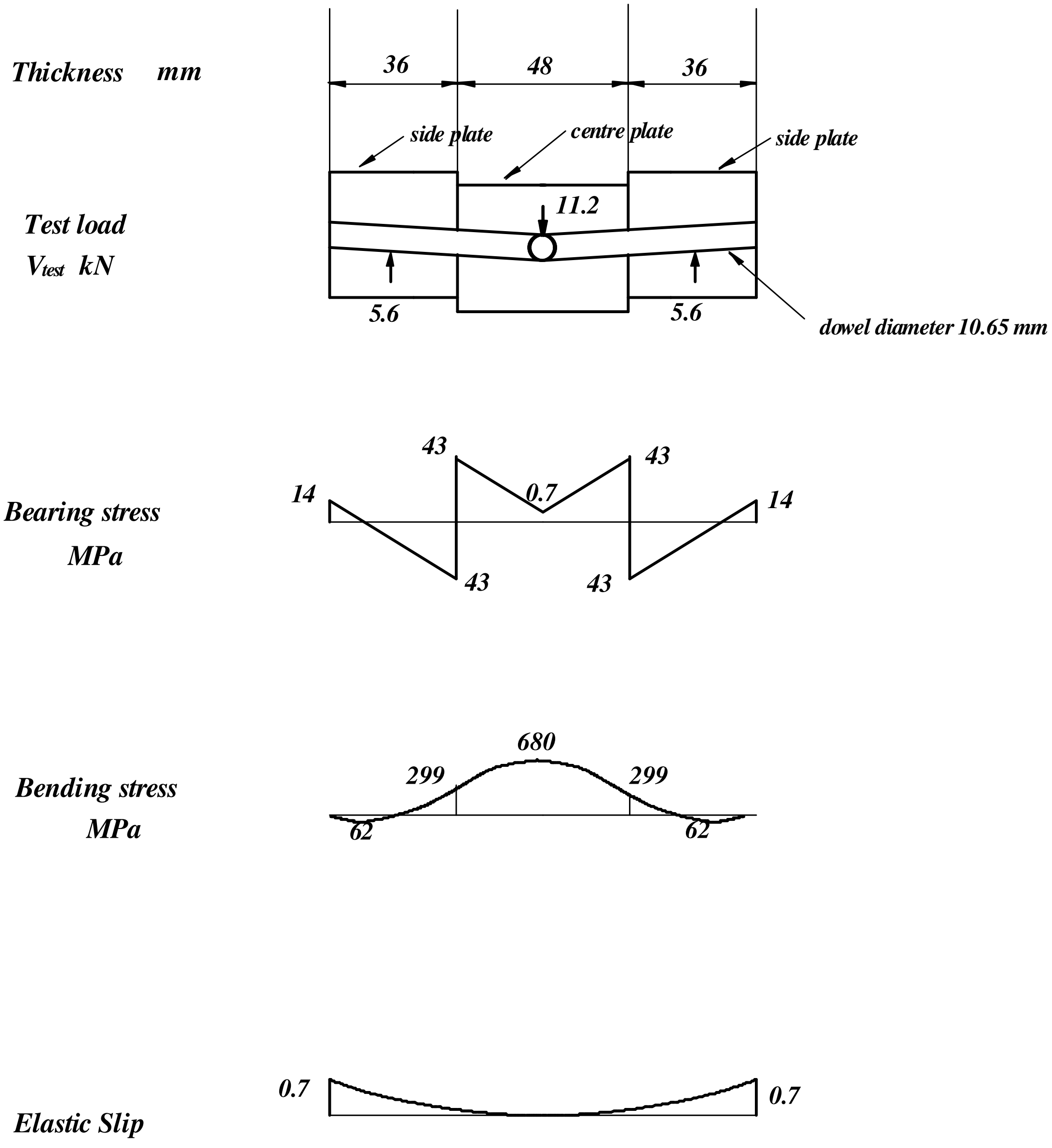


Figure 6 : Analysis of Series 4

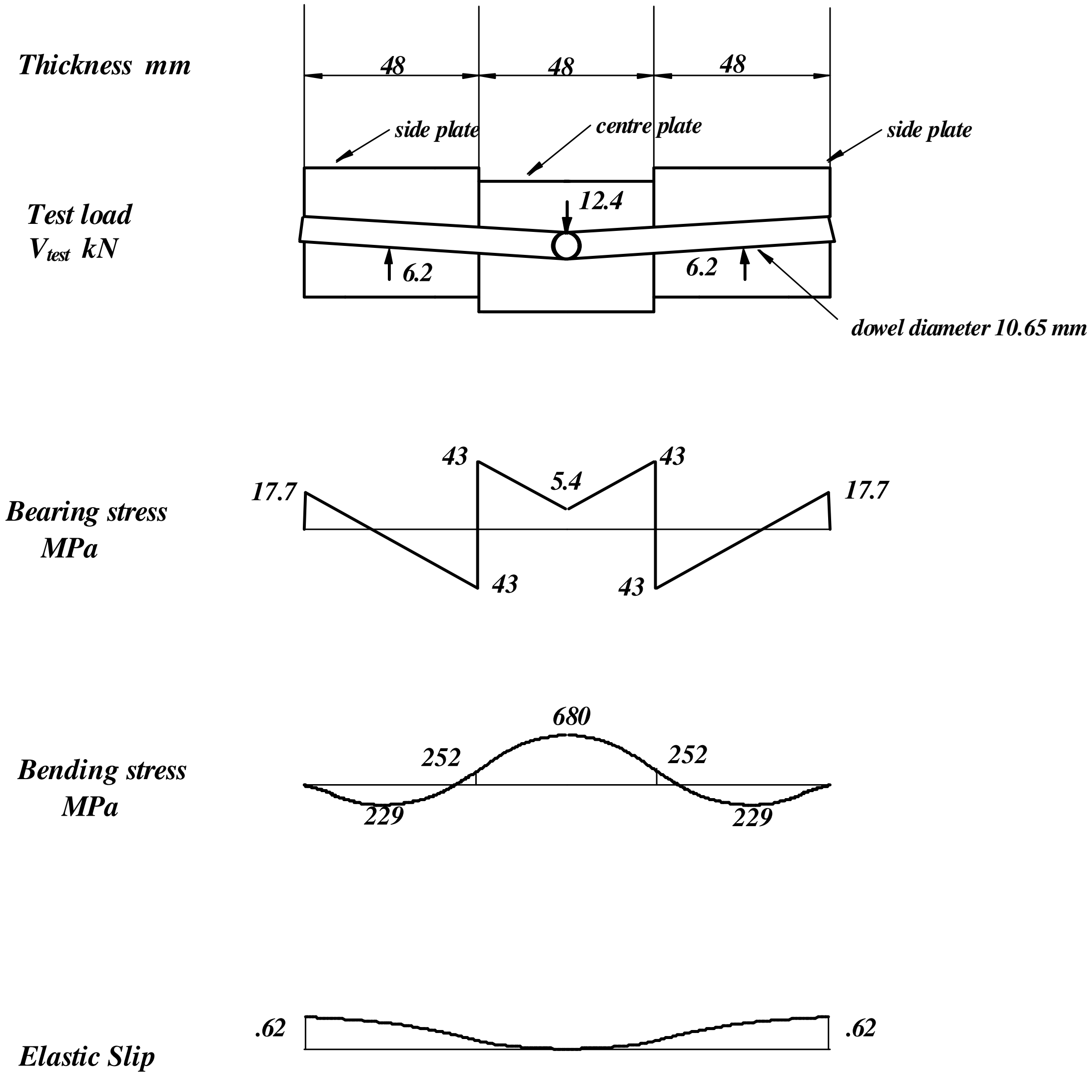


Figure 7 : Analysis of Series 5

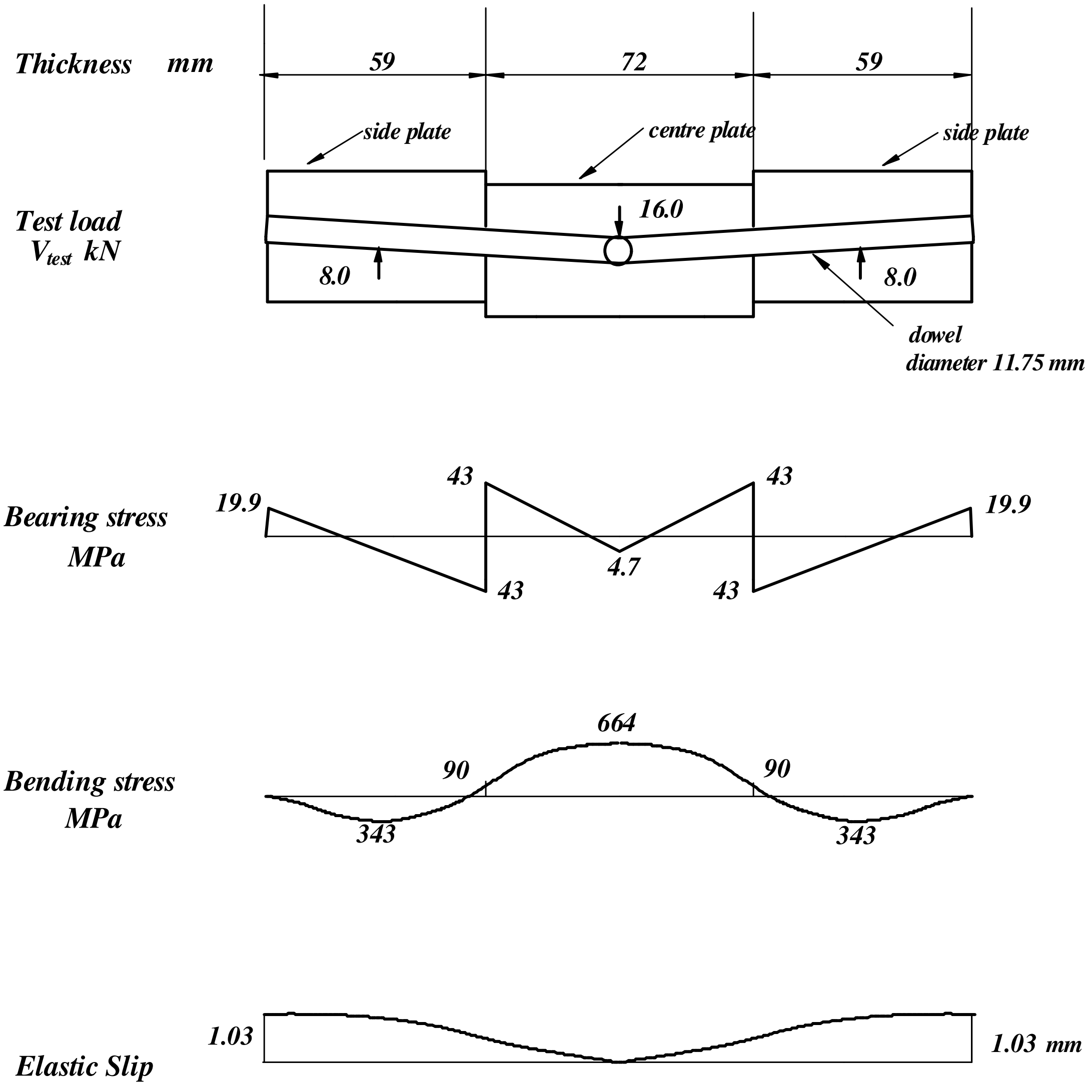
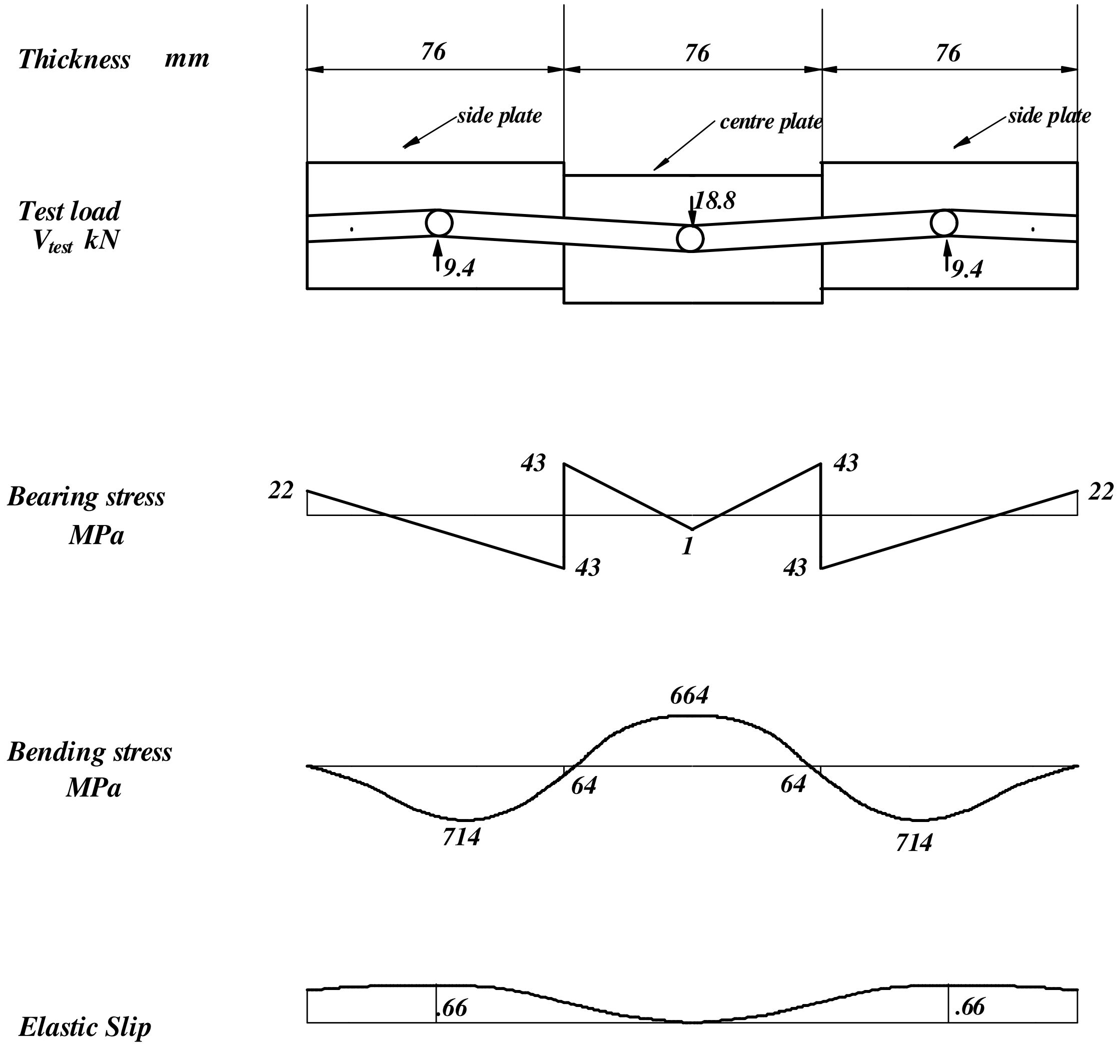


Figure 8 : Analysis of *EDM* critical embedment



Conclusions

The ***EYM*** underestimates the critical embedment length by 22% and the potential resistance of a dowelled connection by 6%.

The ***EDM*** equations give a value for resistance which is 2% higher than that found by computer analysis.

References

1.. The Mathematics and Computer Modeling of Nailed Shear Connections.
Analysis, computer modeling and resistance at ultimate load.
Colin Nicol Smith , October 2015. www.nicolsmith.com.

2. Double shear timber connections with dowel type fasteners.
Doctoral Thesis by Andre Jorissen, 1999

Date : 30th March 2018