The mathematics and computer modelling of nailed shear connections

Introduction

During the Second World War a Swedish researcher into the behavior of reinforced concrete, Dr.K.W. Johansen was deprived of raw materials so he turned his attention to a less pressing but nonetheless intriguing problem : the interaction of crushing and bending in dowelled shear connections in wood.

The result was a mathematically rigorous series of equations based upon an erroneous assumption.

The uniform distribution of bearing stress along the dowel assumed by Johansen combined with a uniform bearing test made a comprehensive series of the yield models possible. The so called European Yield Model has been adopted by timber codes throughout Europe and North America, but the uniform distribution of compressive load underestimates the critical embedment length and cannot be used to analyse a dowel connection.

In explaining the interaction of crushing of the wood and bending of the dowel correct determination of the shape of the loading curve is essential particularly with the use of self tapping screws (STS) in cross laminated timber (CLT) construction.

The actual distribution of compressive load along a dowel has been verified by experiment.

The characteristic curve of joint slip against shear load (the only factors which can be measured directly in a test) can be modeled by assuming that the distribution follows an exponential curve. Successive integration of the equation for the loading curve gives shear, bending, and deflection curves, the latter duplicating the shape of the loading curve.

In other words the deflection of any point is proportional to the compressive load at that point.

Application of this theory a) duplicates the test curves b) enumerates the crushing stress and c) correctly predicts the formation and location of plastic hinges in the dowel.

Exponential Distribution Model (EDM)

Figure 1 shows the classic layout of beam theory applied to two similar blocks of wood connected by a dowel of diameter d. Load intensity, shear force, bending moment and deflection can all be derived by integration. The only unusual thing about this diagram is that the load intensity is represented by a line following an exponential curve, the load intensity p being equated with a function containing A, B, α and C. A is required for scale. B gives the curvature of the line and C is required for the reverse pressure which is required for resolution of the statics. α gives the portion of the length L to the point of where the load intensity is being determined. At $\alpha = 1$ the load intensity is a maximum at the shear interface. Load intensity at this point, $p_{\max}$, the maximum load on the dowel is equal to the maximum bearing stress multiplied by the diameter of the dowel.

The shear at the interface is given by $V_{\max}$, and where the shear is equal to zero, maximum bending occurs, the distance to the point of maximum bending being given by kL. The deflection of the dowel is given by δ and, as can be seen, the shape of the deflected curve closely resembles the shape of the load intensity diagram. It is important to note that the quantity δ as calculated will be the deflection of the dowel on one side of the connection. In other words the deflection is calculated for an element and the joint slip is equal to twice δ in the case of a symmetrical connection.

The Exponential Distribution Model was first suggested in a paper by Nicol- Smith (1971).

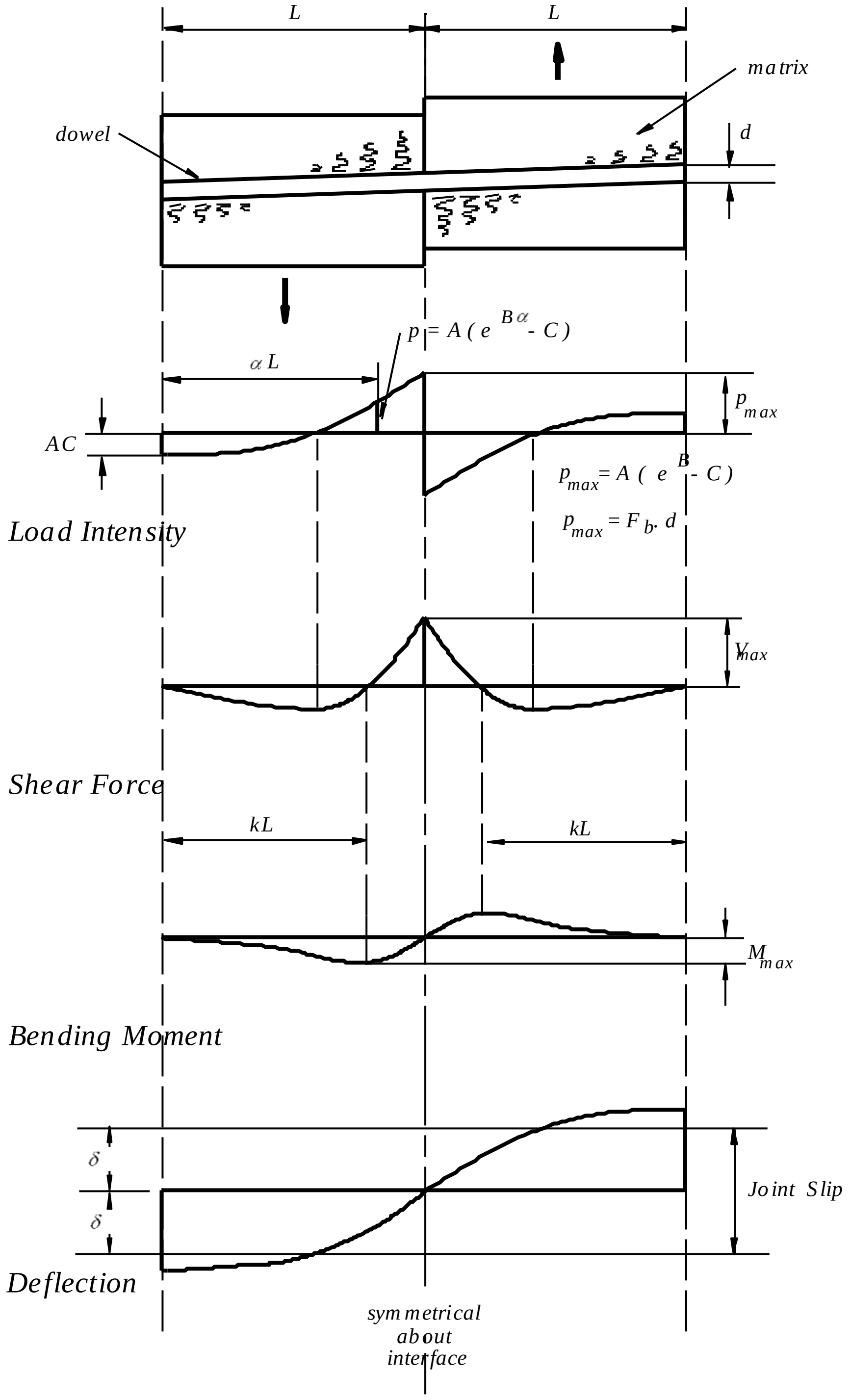

L
L
matrix
dowel
d
$p = A(e^{B\alpha} - C)$
αL
AC
p_{max}
$p_{max} = A(e^{B} - C)$
$p_{max} = F_b \cdot d$
Load Intensity
V_{max}
Shear Force
kL
kL
M_{max}
Bending Moment
δ
δ
Joint Slip
Deflection
symmetrical
about
interface

Figure 1

Application of the EDM.

Successive integration of the load intensity diagram results in complicated expressions all of which are functions of B and resolving these expressions results in useful equations which can be used to determine shear, bending, and deflection in terms of bearing stress, diameter, length and a co-efficient which varies depending upon the value of B. The co-efficients are K_3, K_4 and K_6 applied respectively to the equations for shear, bending, and deflection.

$$V = \frac{F_b d L}{K_3} \qquad (1)$$

$$M = \frac{F_b d L^2}{K_4} \qquad (2)$$

$$\delta = \frac{F_b d L^4}{K_6 E I} \qquad (3)$$

The next step is the derivation of values for the co-efficients. This was accomplished using a computer and made allowance for the clamping effect at the end of the dowel.

The resulting tabulation is given in Table 1.

B	K_3	K_4	K_6
6	8.5	89	595
5	7.6	81	460
4	6.7	66	345
3	5.9	53	245
2	5.2	42	164
1	4.5	33	111
0	4.0	27	54

Table 1

Co-efficients for the EDM equations

The EDM provides a means of determining bearing pressure and bending moment in the dowel when only the shear and deflection can be directly measured experimentally.

B is a measure of the curvature of the bearing pressure distribution. It is counter-intuitive. High curvature occurs at low load and decreases to the point where the dowel is straight at $B = 0$ and load transfer to the wood is maximized.

Test

Small diameter nails connecting two similar blocks of wood in shear were tested in groups of four with varying embedments ; 11mm, 22mm, and 38mm, the nails being 2.84mm in diameter in pre-bored spruce specimens, gapped to eliminate friction. Deflections, as could be expected, varied but when averaged gave the load/deflection curves shown in

Figure 2.

The yield stress of the nails F_y was established in a separate test.

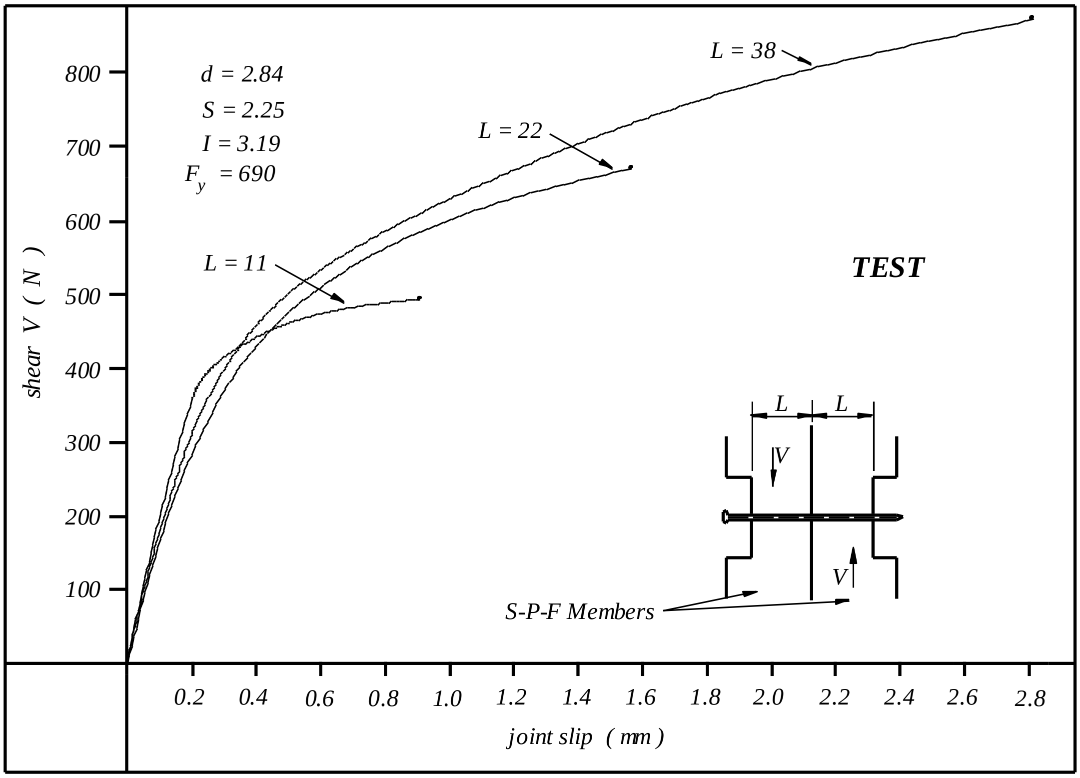


Figure 2

The equations of the EDM were used to construct theoretical curves, shown in Figure 3 on which the values of the B were used progressively in descending order to show how deflection increased as load was applied to the joint. The values of B are shown on the curves .

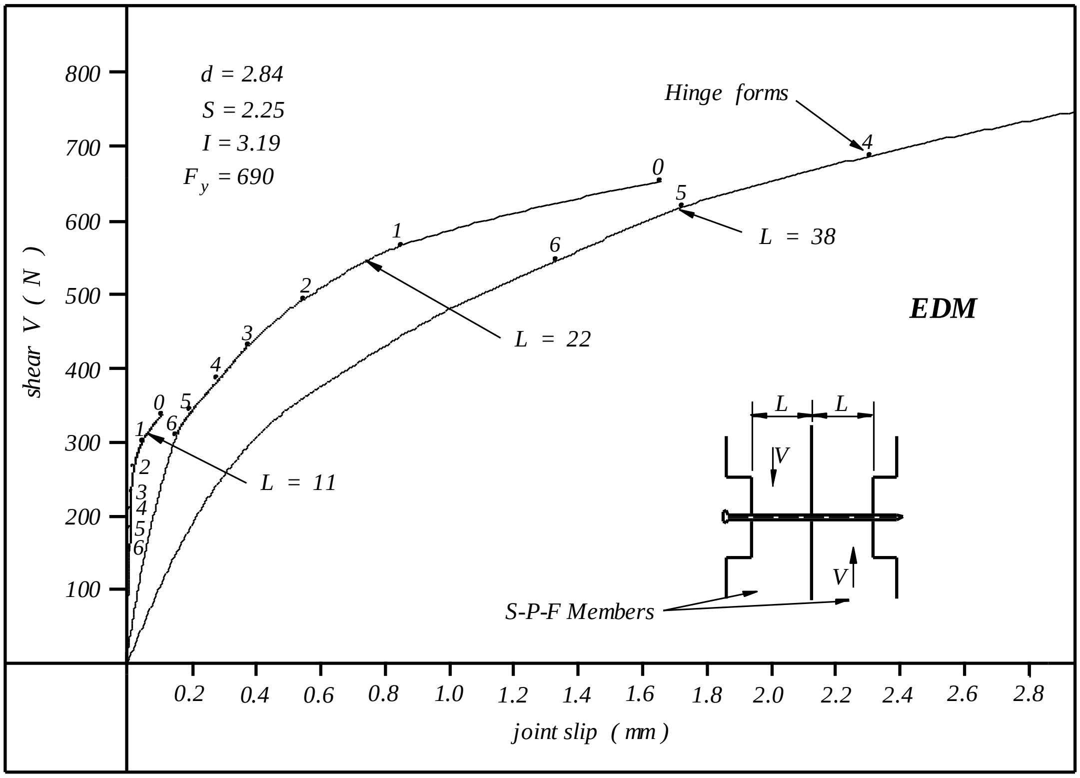


Figure 3

The only value for the bearing pressure which replicated the test curves when the EDM equations linking V and δ were applied was 43 MPa, henceforth denoted in this paper as F_c the crushing strength.

The yield stress in bending for a box nail of 2.84mm diameter is 690 MPa (100 Ksi). Bending stress at failure ie. at hinge formation is 690 MPa multiplied by 1.7 = 1173 MPa

The curve for L = 38mm shows a hinge forming at a shear of 693 N and joint slip of 2.31 mm when $B = 4$, at which point the bending stress is

1187 MPa.

The curve for L = 11mm does not match the test curve although the (problematic) value of $K_6 = 55$ at $B = 0$ seems to give a good maximum value for shear of 334 N and joint slip of 0.10 mm.

The more likely scenario is the bearing stress increasing uniformly from zero to a maximum of 43 MPa at failure.

The curve for L = 22mm follows the test curve most closely and indicates a failure load of 647 N at $B = 0$,with a bending stress of 973 MPa at a joint slip of 1.66 mm. Only one of the four 22mm embedded nails developed a hinge during the test. And that was at a failure load 20 % higher than the average.

At B = 0 the load intensity diagram is as shown in Figure 4.

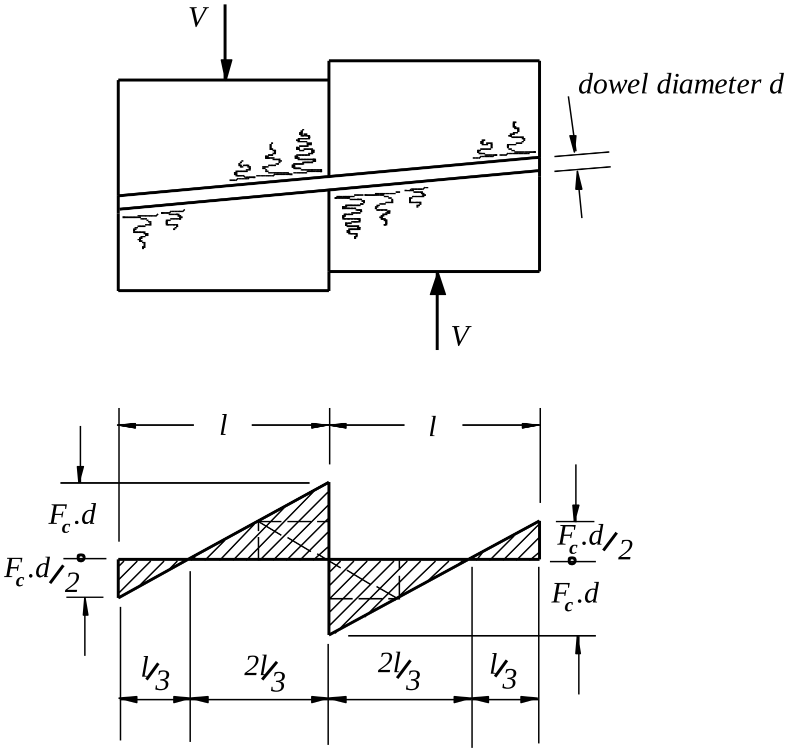


Figure 4

From Table 1 the shear $V = \dfrac{F_c \times d \times l}{4}$

And the maximum bending moment in the dowel $M = \dfrac{F_c \times d \times l^2}{27}$

If M is equated to the ultimate flexural yield moment of the dowel $\dfrac{d^3}{6} \times F_y$

the critical embedment length $l = 2.12\sqrt{\dfrac{F_y}{F_c}} \times d$, and substituting for l

the ultimate shear , or "resistance" $V_{ult} = .53\sqrt{F_y \times F_c} \times d^2$.

In which : d = dowel diameter

F_y = dowel yield stress

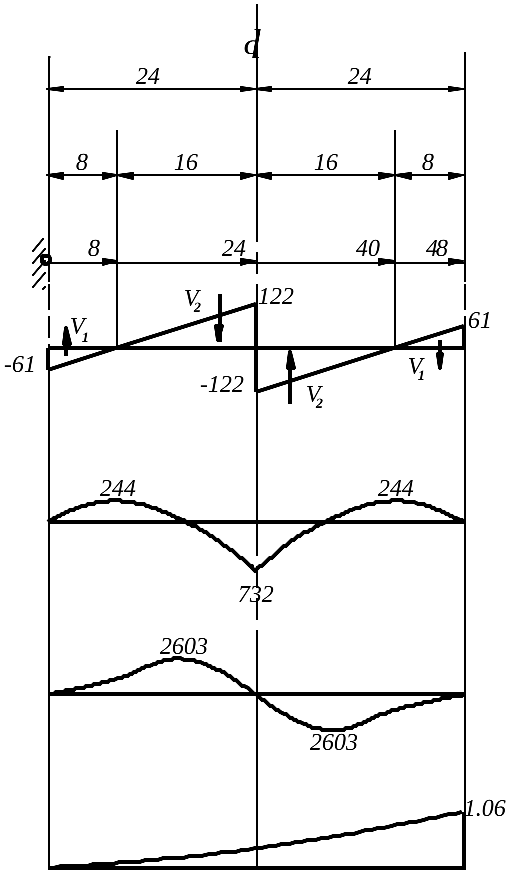
F_c = wood crushing strength

Embedments less than critical will reduce the resistance in direct proportion to the embedment length while embedments more than critical cannot increase the resistance beyond that determined at critical embedment.

Computer Analysis

There is no need to develop the triangular distribution into a comprehensive series of yield models as it is conducive to an extremely simple analytic procedure. Analysis of dowel behavior and failure is extremely simple with a computer beam program . As long as the distribution of load along the beam is known. Use of a triangular distribution with crushing strength at the interface makes this possible for all types of connections.

Considering the same small nails used in the test the ultimate behaviour canbe analysed as follows, starting with critical embedment in a symmetrical connection as shown in Figure5.



$$V = .53\sqrt{690 \times 43} \times 2.84^2$$

$$= 732$$

$$l = 2.12\sqrt{\frac{690}{43}} \times 2.84$$

$$= 24$$

$$p_2 = 2.84 \times 43$$

$$= 122$$

$$p_1 = 61$$

$$I = 3.19$$

$$E = 200{,}000$$

$$S = 2.28$$

$$F_b = 2603/2.28$$

$$= 1141$$

Figure 5

The load slip is under-estimated as the beam program assumes clamping at the left hand support of the assumed cantilever.

If the embedment is increased V_2 and V_1 will decrease but V cannot increase above *732 N* . By trial and error reduce length *16 mm* in Figure 5 to *14 mm* and p_1 to *11 N /mm* in Figure 6

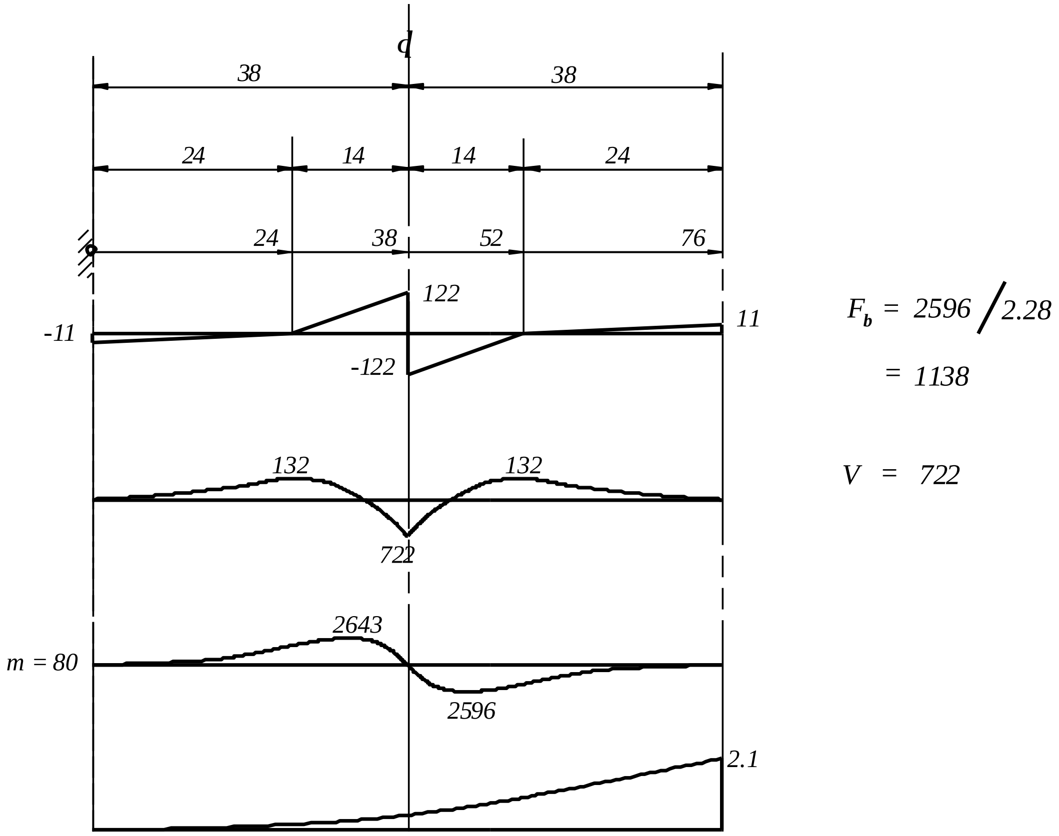
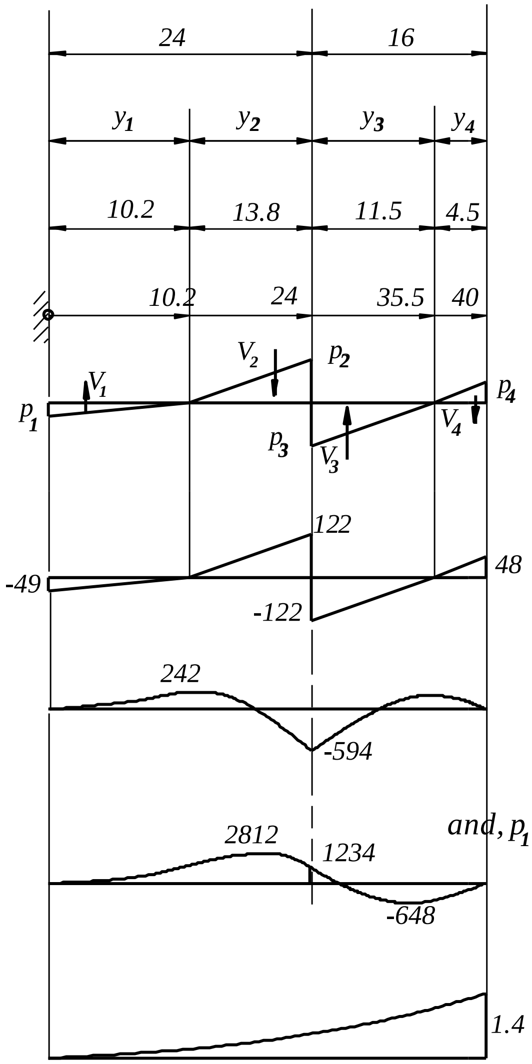


Figure 6.

Typically this is as close as computer beam analysis allows, and because the clamping effect is reduced, load slip in this case is close to that observed in the test. It also simulates the load distribution at $B = 4$.

A similar process is used when dealing with an unsymmetrical connection as shown in Figure 7.



$$\text{As before: } p_2 = p_3 = 122$$

$$r = \sqrt{\frac{16}{24}} = .816$$

$$V = .816 \times 732 = 598$$

$$y_4 = \left(\frac{16^2}{2} - \frac{598 \times 16}{122}\right) \Big/ \left(16 - \frac{598}{122}\right)$$

$$= 4.5$$

$$p_4 = 122 \times 4.5 / 11.5$$

$$= 48$$

$$X = y_2 / y_3$$

$$\text{try } X = 1.2$$

$$y_2 = 1.2 \times 11.5$$

$$= 13.8$$

$$\text{and, } p_1 = \frac{2 \times 598\,(1.2 - 1) + 1.2 \times 48 \times 4.5}{24 - 1.2 \times 11.5}$$

$$= 49$$

$$F_b = 2812 / 2.28$$

$$= 1233$$

Figure 7.

Having manipulated the load triangles to satisfy static equilibrium, the computed bending stress is found to be acceptable and hence the square root reduction factor for side member load is justified. Guesswork is minimized by use of the expressions for y_4 and p_1.

Conclusions

Although the test results cited in this paper apply to nails, studies at various load levels relate the EDM to the resistance of Griplam Rivets, Bolts, Lag Screws, and Dowels in Double Shear Connections.

The crushing strength should be reduced to a 5% exclusion limit to allow for weaker wood and further reduced to allow for duration of load. Both these effects will result in increased critical embedment length and should be considered in preparing the EDM for Code use. Otherwise the expression for resistance based upon crushing strength and yield strength is an ideal for Code use when used in conjunction with appropriate reduction factors for embedment lengths less than critical. The emphasis however in a code should be upon guidance in providing adequate embedment and encouraging the use of efficient connections.

Acknowledgements

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List of Symbols		**units**
A	*Scale coefficient in EDM.*	-
B	*Curvature coefficient in EDM.*	-
C	*Reverse pressure coefficient in EDM.*	-
d	*Diameter of dowel*	*mm.*
e	*Base for natural logarithms*	-
E	*Modulus of Elasticity of dowel*	*MPa*
EDM	*Exponential Distribution Model*	
EYM	*European Yield Model*	
F_b	*Bearing Stress (Figure 1)*	*MPa*
F_b	*Dowel Bending Stress (Examples)*	*MPa*
F_c	*Wood Crushing Stress*	*MPa*
F_y	*Yield Stress in dowel*	*MPa*
I	*Moment of Inertia of dowel*	mm^4
k	*Fraction of embedded length to point of maximum bending*	-
K	*Embedment Factor*	-

K_1 , K_2	*Configuration coefficients*	-
K_3,K_4	*Coefficients in EDM equations*	-
K_6	*Deflection Coefficient in EDM*	-
l	*Critical embedment length*	*mm*
L	*Actual embedment length*	*mm*
M	*Dowel Bending Moment*	*N.mm*
p	*Load intensity on dowel*	*N per mm*
r	*Reduction Coefficient*	-
S	*Section modulus of dowel*	mm^3
V	*Shear at interface*	*N , kN*
α	*Fraction of embedded length in EDM*	-
δ	*Dowel Deflection*	*mm*

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